AP Calculus AB **3.1 – 3.3 AP Review Questions** **Name** ________________________

1.

t Hours	0	1	3	6	8
R(t) Liters/hour	1340	1190	950	740	700

Water is removed from a tank at a rate modeled by R(t) liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of R(t) are shown in the table above. Estimate $R'(2)$. Show your work that leads to your answer. Indicate the appropriate units.

$\frac{R(3)-R(1)}{3-1} = \frac{950-1190}{2} = \frac{-240}{2} = -120$ liters / hour^2

2. (no calculator)

Let $f(x) = 2x^3 - x$ and $g(x) = \frac{f(x)}{x}$.

(a) What is the slope of the graph of f at x = -1? Show the work that leads to your answer.

$f'(x) = 6x^2 - 1$

$f'(-1) = 6(-1)^2 - 1 = 5$

(b) Write an equation of the line tangent to the graph of g at x = -1.

(Quotient Rule)

Point $g(-1) = \frac{2(-1)^3 - (-1)}{-1}$

$g(-1) = 1$

$(-1, 1)$

Slope $g'(x) = \frac{x(6x^2-1) - (2x^3 - x)(1)}{x^2}$

$= \frac{6x^3 - x - 2x^3 + x}{x^2} = \frac{4x^3}{x^2} = 4x$

$g'(-1) = 4(-1) = -4$

$\boxed{y - 1 = -4(x+1)}$

(c) What is the slope of the line normal to the graph of g at x = -1?

$\perp$ slope $= \boxed{\frac{1}{4}}$

3. Evaluate each limit analytically. Note: Finding the answer should not involve a lengthy algebraic process.

a. $\lim_{h \to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$

$f(x) = \sqrt[3]{x} = x^{1/3}$

$\boxed{f'(x) = \frac{1}{3}x^{-2/3}}$

b. $\lim_{h \to 0} \frac{\sqrt{16+h} - 4}{h}$

$f(x) = \sqrt{x}$ $x = 16$

$f'(x) = \frac{1}{2}x^{-1/2}$

$\boxed{f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}}$

c. $\lim_{h \to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$

$f(x) = \frac{1}{x}$ @ $x = 5$

$f'(x) = -\frac{1}{x^2}$ $f'(5) = \frac{-1}{5^2}$

$= -1/25$

4. Given:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	−3	1	5	−2
5	4	7	−1	2

(a) If $h(x) = \dfrac{f(x)}{g(x)}$, find $h'(2)$.

$$h'(x) = \frac{g(x)\cdot f'(x) - f(x)\ \ '(x)}{[g(x)]^2}$$

$$h'(2) = \frac{g(2)\cdot f'(2) - f(2)g\ \ 2)}{[g(2)]^2} = \frac{5\cdot 1 - (-3)\cdot(-2)}{5^2} = \frac{5-6}{25} = \boxed{\frac{-1}{25}}$$

(b) If $j(x) = f(x) \bullet g(x)$, find j'(5).

$j'(x) = f(x)\cdot g'(x) + g(x)\cdot f\ \ )$

$j'(5) = f(5)\cdot g'(5) + g(5)\cdot -\ (5)$

$= 4\cdot 2 + (-1)\cdot (7)$

$= 8 - 7 = \boxed{1}$

5. (no calculator)

Given: $f(x) = x^2$

(a) Find the slope of the normal line to the graph of f at $x = -3$.

(b) Two lines passing through the point (3, 8) will be tangent to the graph of f. Find an equation for each of these lines.

a) $f'(x) = 2x$

$f'(-3) = 2(-3) = -6$

$\perp m = \boxed{\frac{1}{6}}$

$2x = \dfrac{x^2-8}{x-3}$ slope between (3,8) and (x, x^2) ← f(x)

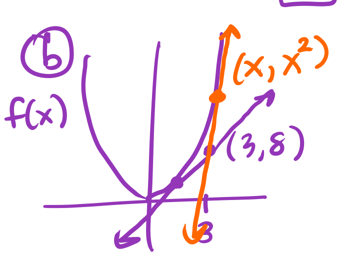
$2x^2 - 6x = x^2 - 8$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2, 4$

(b)



$f'(x) = 2x =$ slope of tangent lines

Point (2,4) Slope $f'(2) = 4$ $\boxed{y - 4 = 4(x-2)}$

Point (4,16) slope $f'(4) = 8$ $\boxed{y - 16 = 8(x-4)}$