8.3 Day 2

Tuesday, March 14, 2017 7:56 AM

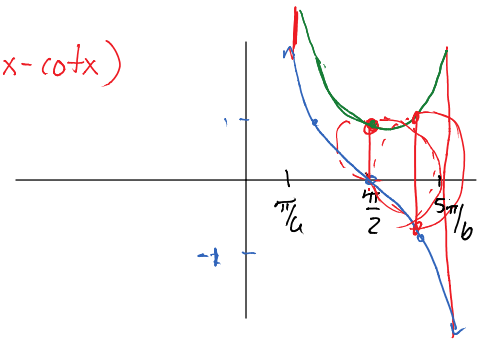
Volume

Ex: Find the volume of the solid that lies between the planes $\perp$ to the x-axis at $x = \frac{5\pi}{6}$ and $x = \frac{\pi}{6}$. The cross-sections are $\perp$ to the x-axis are circular disks with diameters running from the curve $y = \cot x$ to the curve $y = \csc x$.

hint: $\cot^2 x = \csc^2 x - 1$

$d = \csc x - \cot x$

$r = \frac{1}{2}(\csc x - \cot x)$



$A_{cs} = \pi r^2$

$= \pi\left(\frac{1}{2}(\csc x - \cot x)\right)^2$

$= \frac{\pi}{4}(\csc x - \cot x)(\csc x - \cot x)$

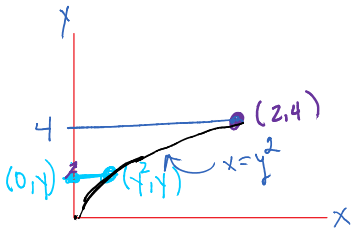
$= \frac{\pi}{4}\left[\csc^2 x - 2\cot x \csc x + \cot^2 x\right]$

$= \frac{\pi}{4}\left[\csc^2 x - 2\cot x \csc x + \csc^2 x - 1\right]$

$V = \int_{x=\frac{\pi}{6}}^{x=\frac{5\pi}{6}} \frac{\pi}{4}\left(2\csc^2 x - 2\cot x \csc x - 1\right) dx$

$V = \frac{\pi}{4}\left[-2\cot x + 2\csc x - x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

Ex 2: Find the volume of the solid $\perp$ to the y-axis at $y = 0$ and $y = 4$. The cross-sections are $\perp$ to the y-axis are isosceles right triangles with the one leg running from the y-axis to the parabola $x = y^2$.



$\ell = y^2 - 0 = y^2$

$A_{cs} = \frac{1}{2}bh$

$= \frac{1}{2}\ell^2$

$= \frac{1}{2}(y^2)^2 = \frac{1}{2}y^4$

$V = \int_{y=0}^{y=4} \frac{1}{2}y^4\, dy$

$= \frac{1}{2}\cdot\frac{y^5}{5}\Big|_0^4$

$= \frac{4^5}{10} = \frac{1024}{10}$ ★