

Find the limit algebraically. No calculators.

1. $\lim_{x \rightarrow 4} \frac{4-x}{x-1} = \frac{4-4}{4-1} = \frac{0}{3} = \boxed{0}$ 2. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{4x}{\tan x} = \frac{4(\frac{\pi}{4})}{\tan(\frac{\pi}{4})} = \frac{\pi}{1} = \boxed{\pi}$ 3. $\lim_{x \rightarrow 0} \frac{1-\cos x}{\sin x} = \frac{1-\cos 0}{\sin 0} = \frac{0}{0}$

$\lim_{x \rightarrow 0} \frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x \rightarrow 0} \frac{1-\cos^2 x}{\sin x(1+\cos x)}$
 $= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin x(1+\cos x)} = \lim_{x \rightarrow 0} \frac{\sin x}{1+\cos x} = \frac{0}{2} = \boxed{0}$

4. $\lim_{x \rightarrow 2} \frac{x^2+2x-8}{x^2-4} = \lim_{x \rightarrow 2} \frac{(x-2)(x+4)}{(x+2)(x-2)} = \lim_{x \rightarrow 2} \frac{x+4}{x+2} = \frac{2+4}{2+2} = \frac{6}{4} = \boxed{\frac{3}{2}}$ 5. $\lim_{x \rightarrow 0} \frac{x}{x^2-x} = \lim_{x \rightarrow 0} \frac{x}{x(x-1)} = \lim_{x \rightarrow 0} \frac{1}{x-1} = \frac{1}{0-1} = \boxed{-1}$

6. $\lim_{x \rightarrow 0} \frac{\sin x}{5x} = \lim_{x \rightarrow 0} \frac{1}{5} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{1}{5} \cdot 1 = \boxed{\frac{1}{5}}$

7. $\lim_{t \rightarrow 0} \frac{\tan^2 t}{t} = \lim_{t \rightarrow 0} \frac{\sin^2 t}{\cos^2 t} \cdot \frac{1}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \lim_{t \rightarrow 0} \sin t \cdot \lim_{t \rightarrow 0} \frac{1}{\cos^2 t} = 1 \cdot 0 \cdot 1 = \boxed{0}$

8. $\lim_{x \rightarrow 10^+} \frac{|x-10|}{x-10} = \frac{|1-10|}{1-10} = \frac{9}{-9} = \boxed{-1}$

9. $\lim_{x \rightarrow -2} \frac{x^3+8}{x+2} = \lim_{x \rightarrow -2} \frac{(x+2)(x^2-2x+4)}{(x+2)} = \lim_{x \rightarrow -2} (x^2-2x+4) = (-2)^2 - 2(-2) + 4 = 4 + 4 + 4 = \boxed{12}$

10. $\lim_{x \rightarrow 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x} = \lim_{x \rightarrow 0} \frac{\frac{3 - (3+x)}{3(3+x)}}{x} = \lim_{x \rightarrow 0} \frac{-x}{3(3+x)} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{-1}{3(3+x)} = \boxed{-\frac{1}{9}}$

11. $\lim_{x \rightarrow 2} \frac{\frac{1}{2} + \frac{1}{x-4}}{x-2} = \lim_{x \rightarrow 2} \frac{\frac{x-4+2}{2(x-4)}}{x-2} = \lim_{x \rightarrow 2} \frac{x-2}{2(x-4)} \cdot \frac{1}{x-2} = \lim_{x \rightarrow 2} \frac{1}{2(x-4)} = \boxed{-\frac{1}{4}}$

12. $\lim_{x \rightarrow -\frac{1}{3}} \frac{3 + \frac{1}{x}}{6x+2} = \lim_{x \rightarrow -\frac{1}{3}} \frac{\frac{3x+1}{x}}{2(3x+1)} = \lim_{x \rightarrow -\frac{1}{3}} \frac{3x+1}{x} \cdot \frac{1}{2(3x+1)} = \lim_{x \rightarrow -\frac{1}{3}} \frac{1}{2x} = \frac{1}{2(-\frac{1}{3})} = \boxed{-\frac{3}{2}}$