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2019

AP® CollegeBoard

AP® Calculus AB

Free-Response Questions

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

   (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.

   (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?

   (c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

   End pts

   (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

$E'(5) - L'(5) \approx -10.723$

The rate of number of fish in pond is decreasing since $E'(5) - L'(5) < 0$

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a. $\int_0^5 E(t)\,dt \approx 153.458 = 153$ fish

b. $\frac{1}{5}\int_0^5 L(t)\,dt \approx 6.059$ about 6 fish per hour

c. Let $T(t)$ = total fish in the lake

$T'(t) = E(t) - L(t)$

$T'(t)$: $+$ on $(0, 6.204)$, $-$ on $(6.204, 8)$; 0 min, 8 min

$T(t) = \int_0^t T'(t)\,dt$

t	$T(t)$
0	0
6.204	135.015
8	8.92

The max amt of fish is ≈ 135 @ $t = 6.204$ hours b/c $E(t) - L(t)$ goes from + to − @ $t = 6.204$.

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

since $v_p(t)$ is continous and differentiable. the mvt applies ∴ there is a least one time $v'p(t)=a_p(t)=0$

* $\frac{V_p(2.8)-V_p(0.3)}{2.8-0.3}=0$

(b) Use a trapezoidal sum with the three subintervals [0, 0.3], [0.3, 1.7], and [1.7, 2.8] to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

$\frac{1}{2}(.3)(55+0)+\frac{1}{2}(1.4)(55+-29)+\frac{1}{2}(1.1)(55+48)=83.1$ meters

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

$t \approx 1.866 = A$ $t \approx 3.519 = B$

$\int_A^B |V_Q t|\,dt \approx 106.109$ m

* Abs. value isn't necessary since $V_Q t > 0$

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

$-90+\int_0^{2.8} V_Q t(dt) \approx 45.938$ meters

distance between 37.162m

Particle Q is to left of particle P

END OF PART A OF SECTION II

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

Circle equation

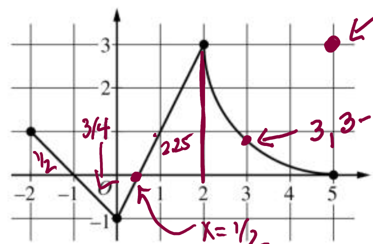
$(x-5)^2+(y-3)^2=9$

$(y-3)^2=9-(x-5)^2$

$y-3=\sqrt{9-(x-5)^2}$

$y=3\pm\sqrt{9-(x-5)^2}$

$y=3-\sqrt{9-(x-5)^2}$ lower part of circle



Graph of f

$y=2x-1$

$9-\frac{1}{4}\pi(9)$

$9-\frac{9\pi}{4}$

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3-\sqrt{5})$ is on the graph of f.

(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

$\int_{-6}^{5} f(x)\,dx - \int_{-2}^{5} f(x)\,dx = \int$

$7-(2+9-9\pi/4)$

$7-11+\frac{9\pi}{4}$ $\quad -4+\frac{9\pi}{4}$

(b) Evaluate $\int_{3}^{5} (2f'(x)+4)\,dx$.

$2\int_3^5 f'(x)\,dx + \int_3^5 4\,dx$

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

$+ \quad - \quad +$

$-2 \quad -1 \quad 1/2 \quad 5$

x	$g(x)$
-2	0
-1	$1/2$
$1/2$	$-1/4$
5	$11-9\pi/4$

$\underline{g'(x)=f(x)}$

(d) Find $\lim_{x\to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$. $= \dfrac{10-3(2)}{1-\pi/4} = \boxed{\dfrac{4}{1-\frac{\pi}{4}}}$

b. $2\left[3-\sqrt{9-(x-5)^2}\right]\Big|_3^5 + 4(5-3)$

$2[0-(3-\sqrt{5})]+8 = 2(-3+\sqrt{5})+8 = -6+2\sqrt{5}+8 = 2+2\sqrt{5}$

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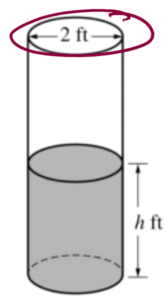

fixed

$\therefore r=1$ is fixed... it will never change no matter the height of water

4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

   $V = 1^2\pi h \quad \frac{dV}{dt} = \pi\frac{dh}{dt} = \pi\left(-\frac{1}{10}\sqrt{4}\right) = -\frac{2}{5}\pi \text{ ft}^3/\text{sec}$

   (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

   $\frac{d^2h}{dt^2} = -\frac{1}{2}\cdot\frac{1}{10}h^{-1/2}\frac{dh}{dt} = \frac{-1}{20\sqrt{h}}\left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$

   $\frac{1}{200} > 0$ the rate of change of height is increasing.

   (c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

   $(0,5)$

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c. $\frac{1}{\sqrt{h}}\,dh = -\frac{1}{10}\,dt$

$\int h^{-1/2}\,dh = \int -\frac{1}{10}\,dt$

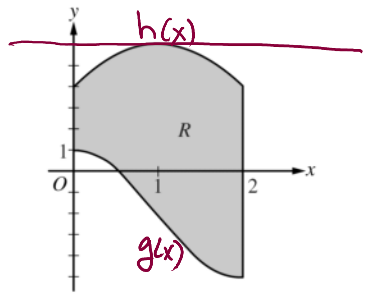
$2h^{1/2} = -\frac{1}{10}t + C$

$h^{1/2} = \frac{-1}{20}t + C$

$\sqrt{5} = C$

$h^{1/2} = -\frac{1}{20}t + \sqrt{5}$

$\boxed{h = \left(-\frac{1}{20}t + \sqrt{5}\right)^2}$



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of R. $\int_0^2 (h(x)-g(x))\,dx = \int_0^2 \left[6-2(x-1)^2-(-2+3\cos\tfrac{\pi}{2}x)\right]dx$

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \dfrac{1}{x+3}$. Find the volume of the solid. $\int_0^2 \frac{1}{x+3}\,dx = \ln|x+3|\Big|_0^2 = \ln 5 - \ln 3$

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

washer

$$\pi\int_0^2 \left[(6-g(x))^2-(6-h(x))^2\right]dx$$

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a. $\int_0^2 \left[8-2(x-1)^2-3\cos\left(\tfrac{\pi}{2}x\right)\right]dx = \left(8x-\tfrac{2}{3}(x-1)^3-\tfrac{6}{\pi}\sin\left(\tfrac{\pi}{2}x\right)\right)\Big|_0^2$

$= 16-\frac{2}{3}-\frac{6}{\pi}-\sin\pi-\left(0+2/3-\frac{6}{\pi}\right) = 16-2/3-6/\pi-2/3+6/\pi$

$16-4/3 = \boxed{44/3}$

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6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

(a) Find $h'(2)$. $= 2/3$

(b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$. $a'(x) = h(x)(9x^2) + 3x^3 \cdot h'(x)$

$a'(2) = h(2)(36) + 24 \cdot h'(2) = 4(36) + 24(2/3) = 144 + 16 = 160$

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim\limits_{x\to2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim\limits_{x\to2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

$1 - (f(2))^3 = 0$ $\quad (f(2))^3 = 1$ $\quad f(2) = 1$

(d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

$$\lim_{x\to2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{2(2)}{-3(f(2))^2 f'(2)} = \frac{4}{-3(1)^2 \cdot f'(2)} = \frac{4}{1}$$

b/c $h(x)$ is twice differentiable it implies continuity $\therefore \lim\limits_{x\to2} h(x) = 4$

STOP

END OF EXAM

$-3f'(2) = 1 \qquad \boxed{f'(2) = -1/3}$

d. $g(x) \leq k(x) \leq h(x) \quad \therefore \quad \lim\limits_{x\to2} g(x) \leq \lim\limits_{x\to2} k(x) \leq \lim\limits_{x\to2} h(x)$

$g(2) \leq k(2) \leq h(2)$

$4 \leq k(2) \leq 4 \qquad 4 \leq \lim\limits_{x\to2} k(x) \leq 4$

so $k(2) = 4$

$\Rightarrow \lim\limits_{x\to2} k(x) = k(2) = 4 \quad \therefore \quad k(x)$ is cont. @ $x = 2$