1. If $y = x\sin x$, then $\frac{dy}{dx} =$

$1 \cdot \sin x + x\cos x$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

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2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

$f'(x) = 300 - 3x^2$

$300 - 3x^2 = 0$

$x^2 = 100$

$x = \pm 10$

f': $-$ | $+$ | $-$

-10 $\quad$ 10

3. $\int \sec x \tan x \, dx =$ $\sec x + C$

(A) $\sec x + C$

(B) $\tan x + C$

(C) $\dfrac{\sec^2 x}{2} + C$

(D) $\dfrac{\tan^2 x}{2} + C$

(E) $\dfrac{\sec^2 x \tan^2 x}{2} + C$

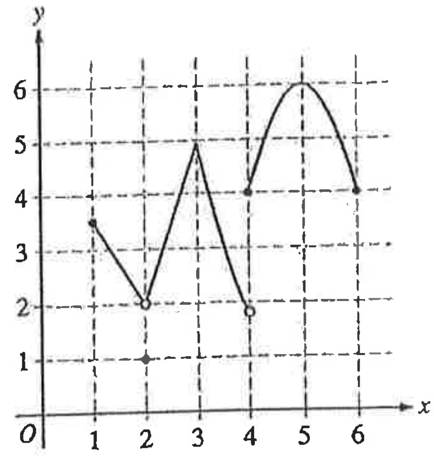
---

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

$f'(x) = 7 + \frac{1}{x}$

$f'(1) = 7 + 1 = 8$



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

(C) $\lim_{x \to 4} f(x)$ exists.

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

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6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

$t(6-t) = 0$

$\int_0^3 (6t - t^2)\,dt$

$= 3t^2 - \frac{1}{3}t^3 \Big|_0^3 = 27 - 9 = 18$

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$ $5(x^3 - \cos x)^4 (3x^2 + \sin x)$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$50 + 3 \cdot 6.2 + 5 \cdot 5.9 + 3 \cdot 5.6$

$= 50 + 18 + .6 + 21 + 4.5 \quad 15 + 1.8$

$= 50 + 18.6 + 29.5 + 16.8$

$= 84.4 + 29.5 = 84.9 + 29 = 114.9$

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$$f(x) = \begin{cases} \dfrac{(2x+1)\cancel{(x-2)}}{\cancel{x-2}} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

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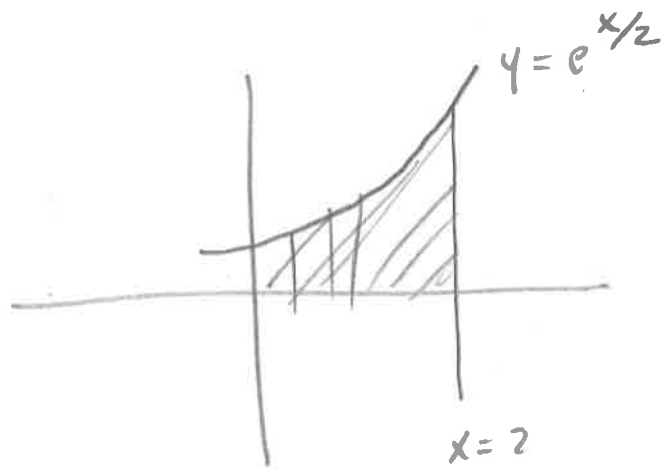
10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$



$$\int_0^2 e^{\frac{x}{2}}\,dx = 2e^{\frac{x}{2}}\Big|_0^2 = 2e - 2$$

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

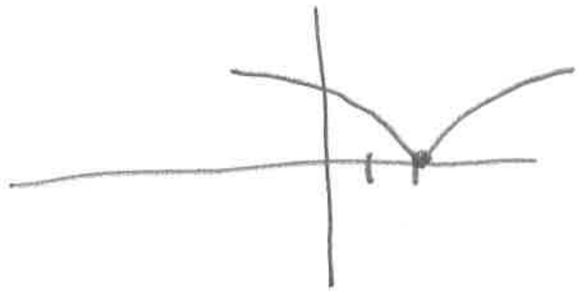
(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.



12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^{4} e^u\,du$ (C) $2\int_1^{2} e^u\,du$ (D) $\frac{1}{2}\int_1^{2} e^u\,du$ (E) $\int_1^{4} e^u\,du$

$u = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-\frac{1}{2}}dx$

$2du = x^{-\frac{1}{2}}dx$

$2\int_1^2 e^u du$

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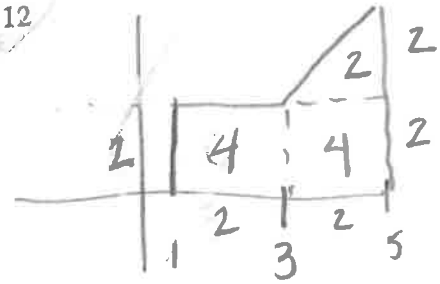
13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12



14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$\cancel{2\tfrac{1}{2}(x^2-4}$$

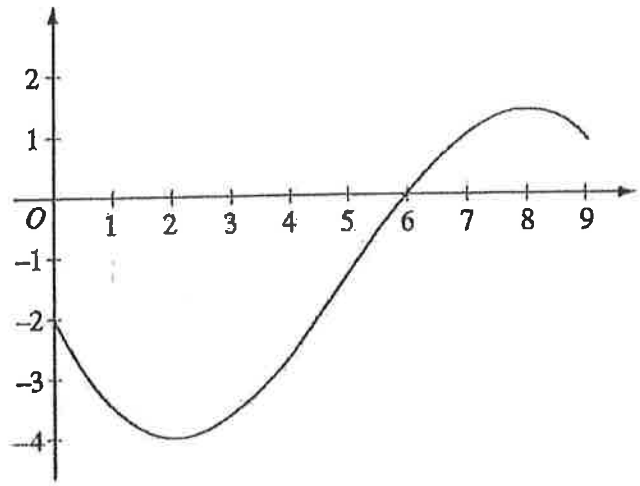
$$f(g(x)) = \sqrt{(3x-2)^2 - 4}$$

$$\frac{d}{dx} f(g(x)) = \frac{1}{2}\left((3x-2)^2 - 4\right)^{-1/2} \cdot \left(2(3x-2) \cdot 3\right)$$

@ $x = 3$

$$= \frac{1}{2}(7^2 - 4)^{-1/2} \cdot (14 \cdot 3)$$

$$= \frac{1}{2\sqrt{45}} \cdot 42 = \frac{21}{3\sqrt{5}} = \frac{7}{\sqrt{5}}$$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

$h(6) = \int_0^6 f(t)\,dt < 0$

$h'(6) = f(6) = 0$

$h''(6) = f'(6) > 0$

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

(A) $t = ab$

(B) $t = \frac{a+b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

(E) $t = a$ and $t = b$

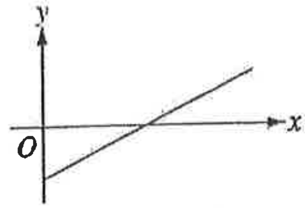
$v(t) = x'(t) = (t-a)\cdot 1 + 1\cdot(t-b)$

$v(t) = 0$

$t - a = -(t-b)$

$2t = b + a$

$t = \frac{a+b}{2}$



Graph of f

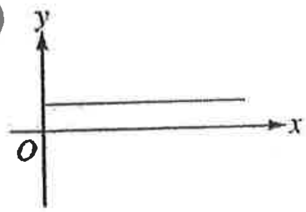
17. The figure above shows the graph of f. If $f(x) = \int_2^x g(t)\,dt$, which of the following could be the graph of $y = g(x)$?

$f'(x) = g(x) = \text{constant}$

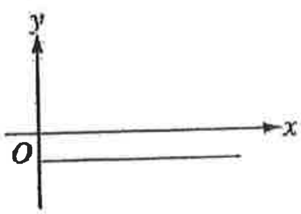
And

$f'(x) > 0$ so $g(x) > 0$

(A)



(B)



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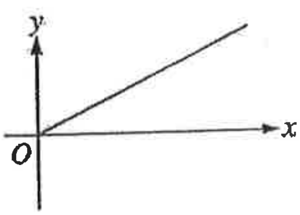
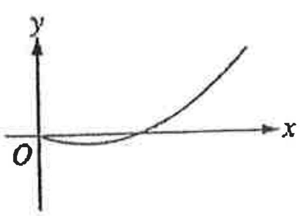
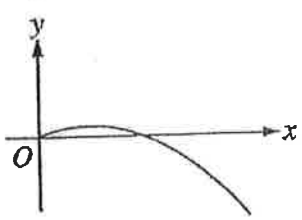


(D)



(E)



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18. $\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h}$ is $= \frac{d}{dx} \ln x \Big|_{x=4} = \frac{1}{x}\Big|_{x=4} = \frac{1}{4}$

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

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19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

$$f'(x) = \frac{(x+2)\cdot 1 - x}{(x+2)^2} = \frac{1}{2}$$

$$\frac{2}{(x+2)^2} = \frac{1}{2}$$

$$(x+2)^2 = 4$$

$$x+2 = \pm 2$$

$$x = \pm\sqrt{2} - 2$$

$$x = 0, -4$$

$$f(0) = 0 \quad f(-4) = 2$$

20. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

$(0,1)$ IS ON $f(x)$

$\therefore (1,0)$ IS ON $f^{-1}(x)$

$f'(x) = 3(2x+1)^2 \cdot 2$

$g'(1) = \frac{1}{f'(0)}$

$= \frac{1}{3(1)^2 \cdot 2} = \frac{1}{6}$

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \cancel{\frac{\sin(5x)}{x}}$ (B) $\cancel{y = 5x}$ (C) $y = \cancel{\frac{1}{x-5}}$ (D) $y = \cancel{\frac{5x}{1-x}}$ (E) $y = \frac{20x^2 - x}{1+4x^2}$

$\lim_{x \to \pm\infty} y = 5$

22. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

Dom $(0,\infty)$

$f'(x) = \frac{x \cdot \frac{1}{x} - 1 \cdot \ln x}{x^2} = \frac{1-\ln x}{x^2}$

$f'(x) = 0$ @ $\ln x = 1$ OR $x = e$

$f(e) = \frac{\ln e}{e} = \frac{1}{e}$

$f'(x)$ e

---

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

$\int dp = \int 200\,dt$

$p = 200t$

24. Let g be the function given by $g(x) = x^2e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

$$g'(x) = 2xe^{kx} + x^2 \cdot ke^{kx}$$

$$g'\left(\tfrac{2}{3}\right) = \frac{4}{3}e^{\frac{2}{3}k} + \frac{4}{9} \cdot ke^{\frac{2}{3}k} = 0$$

$$e^{\frac{2}{3}k}\left(\frac{4}{3} + \frac{4}{9}k\right) = 0$$

$$\frac{4}{3} + \frac{4}{9}k = 0$$

$$\frac{4}{9}k = \frac{-4}{3}$$

$$\frac{9}{4} \cdot \frac{4}{9}k = \frac{-\cancel{4}}{3} \cdot \frac{9}{\cancel{4}}$$

$$k = -3$$

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

$\int dy = \int 2\sin x \, dx$

$y = -2\cos x + C$

$1 = -2\cos(\pi) + C$

$1 = 2 + C$

$-1 = C$

$y = -2\cos x - 1$

26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,dt$. Which of the following must be true on the interval $0 < x < 2$?

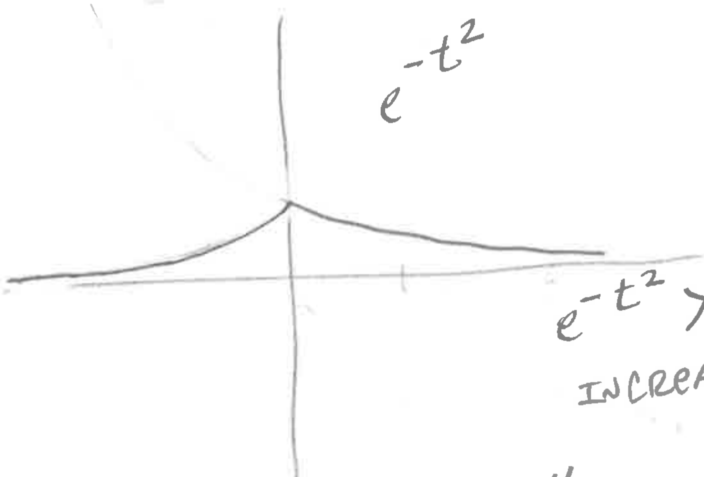
(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.



$e^{-t^2} > 0$ so $\int_0^x e^{-t^2}\,dt$ is increasing

$g''(x) = e^{-x^2}$ does not change sign

$g''(x) > 0$ always

27. If $(x+2y)\cdot\frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

$$\frac{dy}{dx} = \frac{2x-y}{x+2y}$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)(2-y') - (2x-y)(1+2y')}{(x+2y)^2}$$

$$= \frac{\cancel{(3+0)}(\overset{0}{\cancel{2-2}}) - (6-0)(1+4)}{(3+0)^2} = \frac{-30}{9} = \frac{-10}{3}$$

$$\left.\frac{dy}{dx}\right|_{(3,0)} = \frac{6-0}{3+0} = 2$$

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

$$v(t) = x'(t) = \cos(t) + \sin(t) = 0$$

$$\cos t = -\sin t$$

$$t = \frac{3\pi}{4}$$

$$a(t) = x''(t) = -\sin(t) + \cos t$$

$$a\left(\frac{3\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right)$$

$$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -2\frac{\sqrt{2}}{2} = -\sqrt{2}$$