

1. A number plus twice a second number is 108. Find the two numbers that give a maximum product.

$$x + 2y = 108$$

$$y = \frac{108 - x}{2}$$

$$P = xy = x \left(\frac{108 - x}{2} \right)$$

$$P = 54x - \frac{1}{2}x^2$$

$x = 54$ IS A MAXIMUM

$$y = \frac{108 - 54}{2} = 27$$

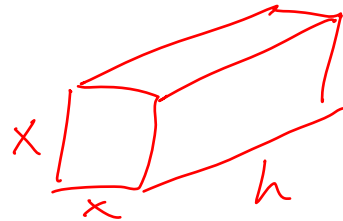
$$x = 54, y = 27$$

2. A rectangular solid with a square base has a surface area of 150 square inches.

- a) Find the maximum volume of the solid and its dimensions.

$$SA = 2x^2 + 4xh = 150$$

$$h = \frac{150 - 2x^2}{4x}$$



$$V = x^2 \cdot h = x^2 \left(\frac{150 - 2x^2}{4x} \right)$$

$$x = 5 \text{ MAXIMIZES } V$$

$$@ x = 5. \quad h = 5$$

$$V_{\max} = x^2 \cdot h = 125 \text{ IN}^3$$

- b) What dimensions can this rectangular solid have if it must have a volume over 100 cubic inches?

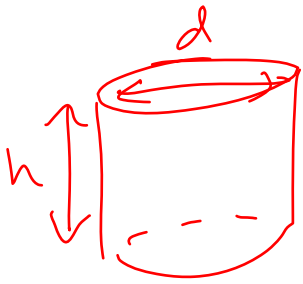
$$100 < x^2 \cdot \left(\frac{150 - 2x^2}{4x} \right) < 125 \quad \text{solve graphically}$$

$$\text{Base: } 3.042 \text{ IN} < x < 6.729 \text{ IN}$$

$$\text{Height: } 2.208 \text{ IN} > \frac{150 - 2x^2}{4x} > 0.806 \text{ IN}$$

3. The diameter plus the height of a cylindrical package is equal to 108 inches.

a) Find the dimensions of the package that gives you a maximum volume.



$$d+h=108$$

$$h=108-d$$

$$V = \pi \left(\frac{d}{2}\right)^2 h$$

$$d = 72 \text{ MAXIMIZES } V$$

$$h = 108 - d = 36 \text{ in}$$

$$V_{\text{MAX}} = 146,574.15 \text{ IN}^3$$

$$V = \pi \frac{d^2}{4} (108-d)$$

b) Find the dimensions of the package if the volume must be over 100,000 cubic inches.

$$100,000 < \pi \frac{d^2}{4} (108-d) < V_{\text{MAX}} \quad \text{solve graphically.}$$

$$\text{Diameter : } 44.931 \text{ IN} < d < 93.407 \text{ IN}$$

$$\text{Height : } 63.069 \text{ IN} < 108-d < 14.593 \text{ IN}$$