2.4 day 1

Monday, August 29, 2016 12:47 PM

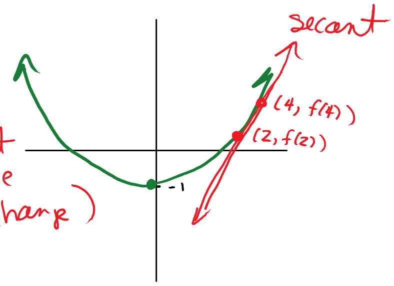
Average Rate of Change:

ex: Find the average rate of change of the function over the given interval.

a. $f(x) = x^2 - 1$ $[2, 4]$

$\frac{f(4) - f(2)}{4 - 2}$ ☆ slope of the secant (Avg. rate of change)

$\frac{15 - 3}{2} = \frac{12}{2} = 6$

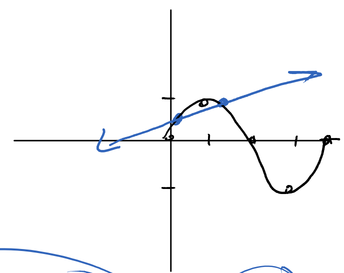


b. $f(x) = \sin x$ $\left[\frac{\pi}{6}, \frac{2\pi}{3}\right]$

$\frac{f\left(\frac{2\pi}{3}\right) - f\left(\frac{\pi}{6}\right)}{\frac{2\pi}{3} - \frac{\pi}{6}}$

$\frac{\sin\frac{2\pi}{3} - \sin\frac{\pi}{6}}{\frac{3\pi}{6}}$ $\frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\frac{\pi}{2}} \Rightarrow \frac{\sqrt{3} - 1}{\cancel{2}} \cdot \frac{\cancel{2}}{\pi}$

$= \frac{\sqrt{3} - 1}{\pi}$

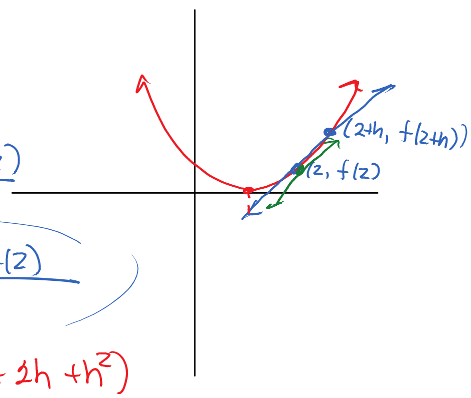


ex $f(x) = (x-1)^2$

find the avg. rate of change $[2, 2+h]$

Avg rate of change (slope of secant line): $\frac{f(2+h) - f(2)}{2+h-2}$

$= \frac{f(2+h) - f(2)}{h}$



$f(2+h) = (2+h-1)^2 = (1+h)^2 = (1 + 2h + h^2)$

$f(2) = (2-1)^2 = 1$

$$\frac{(1+2h+h^2)-1}{h} = \frac{2h+h^2}{h} = \frac{\cancel{h}(2+h)}{\cancel{h}} = 2+h$$

The rate of change @ $x=2$

$$\lim_{h\to 0} \underbrace{\frac{f(2+h)-f(2)}{h}}_{} = \lim_{h\to 0} 2+h = 2+0 = 2$$

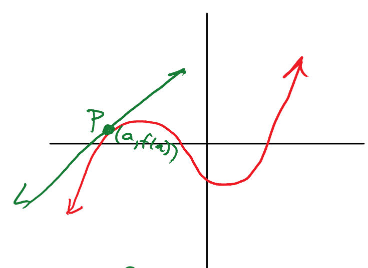
The slope of a function @ a point:

$x=a$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \Rightarrow \text{slope of the tangent line}$$

☆ as long as the limit exist

$(a, f(a))$ point of tangency (p.o.t)



Tangent line: to the curve @ point P (p.o.t) is the line through P with a slope @ P.

Normal line: to the curve @ point P

is perpendicular to the tangent line.

ex: Given $f(x) = x^2 - 4x$

a. write the equation of the tangent line to $f(x)$ @ $x = 1$.

p.o.t. $f(1) = (1)^2 - 4(1) = 1 - 4 = -3$ $\quad f(1) = -3$

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[(1+h)^2 - 4(1+h)] - (-3)}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 4 - 4h + 3}{h}$$

$f(1) = -3$
$m = -2$

$$= \lim_{h \to 0} \frac{-2h + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(-2+h)}{\cancel{h}} = -2 + 0 = -2$$

$$\boxed{y + 3 = -2(x - 1)}$$

b. Write the eqtn. of the normal to $f(x)$ @ $x = 1$.

$$y + 3 = \frac{1}{2}(x - 1)$$

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ex Find the slope of $f(x)$, if it exist, at $x = 0$.

$$f(x) = \begin{cases} 1 + x^2 & x < 0 \quad \text{Left} \\ 2x - 1 & x \geq 0 \quad \text{Right} \end{cases}$$