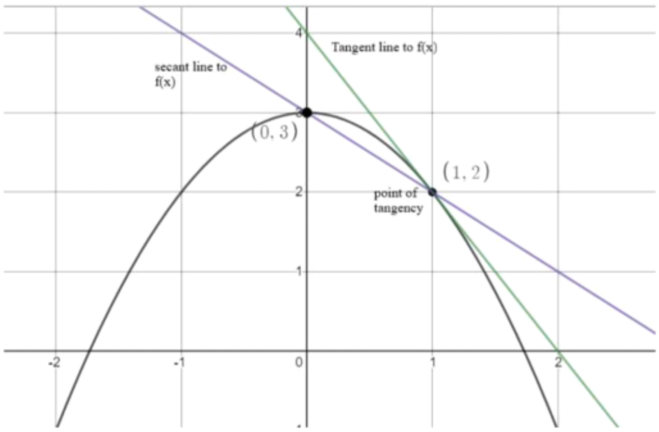
2.4 Day 2 (8/28)

Tuesday, August 27, 2019 3:06 PM

AP Calculus AB
Notes 2.4 Day 2

Name_______________________________________



Use above graph of $f(x) = -x^2 + 3$ to explain the following:

A Point of Tangency (P.O.T):

The relationship between the given secant line and f(x):

The equation of the secant line given above:

The relationship between the given tangent line and f(x):

The equation of the tangent line that has a p.o.t of (1,2):

Write the equation of the normal line that goes through the p.o.t (1,2):

Recall: Write the equations of a tangent and normal lines of $f(x)=1-2x^2$ at x = 2.

$f(2)=-7$ p.o.t

$m=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$

$=\lim_{h\to 0}\frac{1-2(2+h)^2-(-7)}{h}$

$\lim_{h\to 0}\frac{1-2(4+4h+h^2)+7}{h}$

$=\lim_{h\to 0}\frac{1-8-8h-2h^2+7}{h}$

$=\lim_{h\to 0}\frac{-8h-2h^2}{h}$

$\lim_{h\to 0}\frac{\cancel{h}(-8-2h)}{\cancel{h}}$

$=-8-2(0)$

$m=-8$

tangent line $y+7=-8(x-2)$

normal line $y+7=\frac{1}{8}(x-2)$

Determine whether the curve has a tangent line at x = 0.

$$f(x)=\begin{cases}2x, & x<0\\ x^2+2x, & x\geq 0\end{cases}$$

yes!

Left side $m=\lim_{h\to 0}\frac{2(0+h)-0}{h}=\lim_{h\to 0}\frac{2h}{h}=2$

Right $m=\lim_{h\to 0}\frac{(0+h)^2+2(0+h)-0}{h}=\lim_{h\to 0}\frac{h^2+2h}{h}$

$=\lim_{h\to 0}\frac{h(h+2)}{\cancel{h}}=2$

Example:

a. At what point is the slope of the tangent line to $y=x^2-2x+1$ equal to 0.5?

$m=\frac{1}{2}$ (x,y)?

$\frac{1}{2}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\frac{1}{2}=\lim_{h\to 0}\frac{[(x+h)^2-2(x+h)+1]-(x^2-2x+1)}{h}$

$\frac{1}{2}=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{2x}-2h+\cancel{1}-\cancel{x^2}+\cancel{2x}-\cancel{1}}{h}$

$\frac{1}{2}=\lim_{h\to 0}\frac{2xh+h^2-2h}{h}$

$\frac{1}{2}=\lim_{h\to 0}\frac{\cancel{h}(2x+h-2)}{\cancel{h}}$

$\frac{1}{2}=2x-0-2$

$\frac{5}{2}=2x$

$(5/4, 1/16)$

$x=5/4$

$y=\left(\frac{5}{4}\right)^2-2\left(\frac{5}{4}\right)+1$

$=\frac{25}{16}-\frac{5}{2}+1$

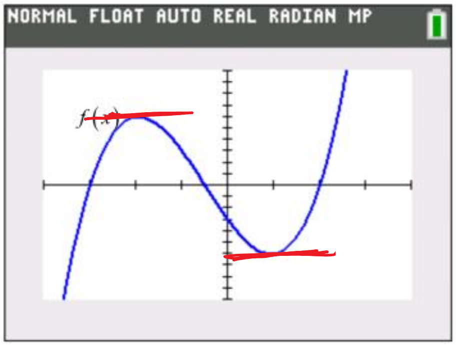
$=\frac{25}{16}-\frac{40}{16}+\frac{16}{16}=1/16$

b. Write the equation of the tangent line using the point from part a.

$m=1/2$ $(5/4, 1/16)$

$y-1/16=\frac{1}{2}(x-5/4)$

Example:



Where would ~~the~~ $f(x)$ have a slope of zero?

Draw the tangent lines on the graph that have a slope of zero.

Example 4: At what **point** is the tangent to $f(x)=3-4x-x^2$ horizontal?

$m=0$

$$0=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

1 2

$$0=\lim_{h\to 0}\frac{[3-4(x+h)-(x+h)^2]-(3-4x-x^2)}{h}$$

$$0=\lim_{h\to 0}\frac{\cancel{3}-\cancel{4x}-4h-\cancel{x^2}-2xh-h^2-\cancel{3}+\cancel{4x}+\cancel{x^2}}{h}$$

$$0=\lim_{h\to 0}\frac{-4h-2xh-h^2}{h} \Rightarrow 0=\lim_{h\to 0}\frac{-\cancel{h}(4+2x+h)}{\cancel{h}}$$

$$0=-4-2x+0$$

$$4=-2x \qquad x=-2$$

$$f(-2)=3-4(-2)-(-2)^2$$
$$=3+8-4$$
$$=7$$

$(-2,7)$ p.o.t