2.1 Day 1

Tuesday, August 13, 2019 10:01 AM

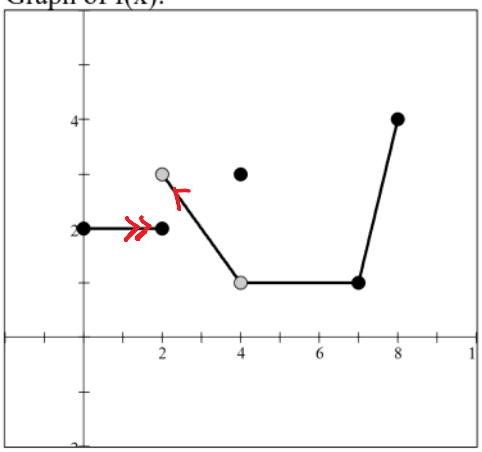
AP Calculus BC
Section 2.1 Notes

Rates of Change and Limits

LIMITS

Limits give us a language for Describing how the outputs (y-values) of a function behave as the inputs (x-value) approach a particular value.

Graph of f(x):



from the left

$\lim_{x\to 2^-} f(x) =$ 2

$\lim_{x\to 2^+} f(x) =$ 3

from the right

$\lim_{x\to 2} f(x) =$ dne

$\lim_{x\to 7^-} f(x) =$ 1

$\lim_{x\to 7^+} f(x) =$ 1

$\lim_{x\to 7} f(x) =$ 1

$\lim_{x\to 4^-} f(x) =$ 1

$\lim_{x\to 4^+} f(x) =$ 1

$\lim_{x\to 4} f(x) =$ 1

$\lim_{x\to 5^-} f(x) =$ 1

$\lim_{x\to 5^+} f(x) =$ 1

$\lim_{x\to 5} f(x) =$ 1

Use a calculator to determine the following limits: $\lim_{x\to\infty}\frac{\sin x}{x} =$ 0 $\quad \lim_{x\to-\infty}\frac{\sin x}{x} =$ 0 $\quad \lim_{x\to 0}\frac{\sin x}{x} =$ 1

Determine the limit by substitution. No Calculators.

1. $\lim_{x\to 2} x^3 - 2x^2 + 3x - 4$

$2^3 - 2(2)^2 + 3(2) - 4$

$= 2$

2. $\lim_{x\to 2}\frac{x^3-1}{x-1}$ $\Rightarrow \frac{2^3-1}{2-1} = \frac{7}{1} = 7$

3. $\lim_{x\to 1}\frac{x^3-1}{x-1}$ $\Rightarrow \frac{1^3-1}{1-1}$ $\frac{0}{0}$

$\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$

$1^2+1+1 = 3$

4. $\lim_{x\to 0}\frac{\tan x}{x}$ $\Rightarrow \frac{0}{0}$

$\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{\boxed{x}} = \lim_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{1}{x}$

$\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}$

$1\cdot 1 = 1$

5. $\lim\limits_{x\to 0}\frac{\sin 4x}{x}$

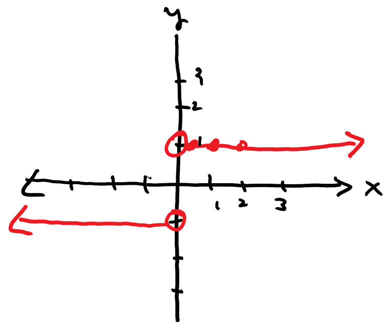
$\lim\limits_{x\to 0}\frac{\sin(ax)}{ax} = 1$

$\lim\limits_{x\to 0} 4\cdot\frac{\sin 4x}{4x}$

$\lim\limits_{x\to 0} 4\cdot \lim\limits_{x\to 0}\frac{\sin 4x}{4x}$

$4\cdot 1 = 4$

6. $\lim\limits_{x\to 0^+}\frac{|x|}{x} = 1$



7. $\lim\limits_{x\to 0}\frac{x+\sin 4x}{x}$

$\lim\limits_{x\to 0}\frac{x}{x} + \lim\limits_{x\to 0}\frac{\sin 4x}{x}$

$\lim\limits_{x\to 0} 1 + \lim\limits_{x\to 0} 4\cdot\frac{\sin 4x}{4x}$ $\quad 1+4 = 5$

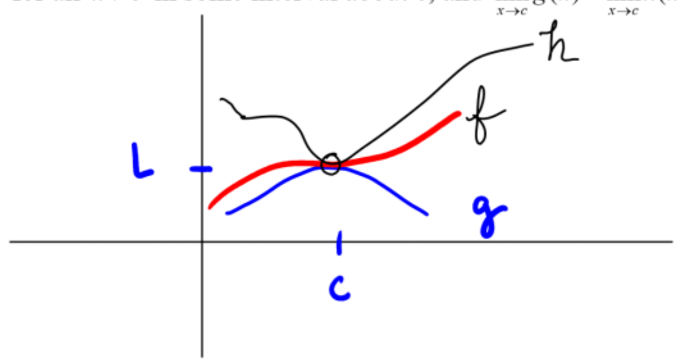
8. $\lim\limits_{x\to 3}\frac{x^2-9}{2x^2-5x-3} = \lim\limits_{x\to 3}\frac{\cancel{(x-3)}(x+3)}{(2x+1)\cancel{(x-3)}}$

$= \frac{3+3}{2(3)+1} = \frac{6}{7}$

The Sandwich Theorem

SQUEEZE

If $g(x)\le f(x)\le h(x)$ for all $x\ne c$ in some interval about c, and $\lim\limits_{x\to c} g(x) = \lim\limits_{x\to c} h(x) = L$ then, $\lim\limits_{x\to c} f(x) = L$



RATES OF CHANGE

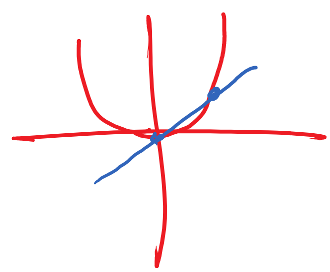
A ball slides down a ramp so that its distance s from the top of the ramp after t seconds is exactly t^2 feet.

t (secs)	s (feet)
0	0
1	1
2	4
3	9
4	16

$S(t) = t^2$

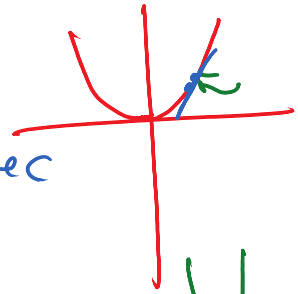
What is the average rate of change (speed) of the ball in the first 3 seconds?

$\frac{S(3)-S(0)}{3-0} = 3\text{ ft/sec}$



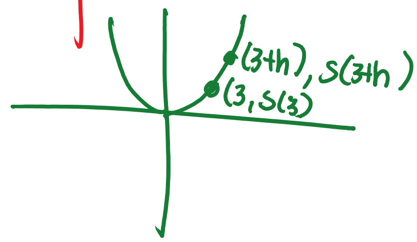
Let's investigate the instantaneous rate of change at 3 seconds.
How fast is the ball going at 3 seconds? Why is the previous answer ruled out? How can a good estimate be found? How can the exact answer be found?

$t = 3.1 \qquad t = 3$



$$\frac{9.61 - 9}{3.1 - 3} = \frac{.61}{.1} = 6.1 \text{ ft/sec}$$

$$\lim_{h \to 0} \frac{s(3+h) - s(3)}{3+h-3}$$



So to find instantaneous rates of change at x = a…

$$\lim_{h \to 0} \frac{s(a+h) - s(a)}{a+h-a} = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$