2.1

Monday, August 15, 2016 1:27 PM

Calc AB 2.1 Name___________________

LIMITS

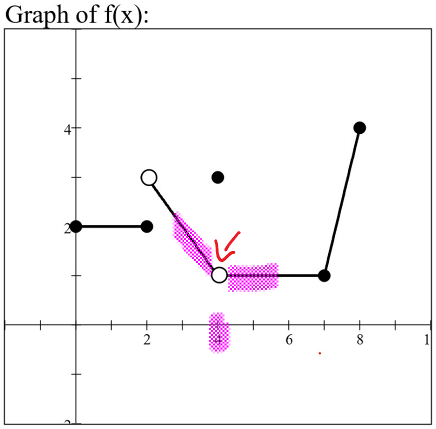
Limits give us a language for describing how the outputs of a function behave as the inputs approach some particular value.

Right-hand limits: $\lim_{x\to c^+} f(x)$ The limit of f as x approaches c from the right

Left-hand limits: $\lim_{x\to c^-} f(x)$ The limit of f as x approaches c from the left

Two-sided limits: $\lim_{x\to c} f(x)$ Exists only if right-hand and left-hand are equal

Graph of f(x):



$\lim_{x\to 2^-} f(x) =$ 2

$\lim_{x\to 2^+} f(x) =$ 3

$\lim_{x\to 2} f(x) =$ dne

$\lim_{x\to 4^-} f(x) =$ 1

$\lim_{x\to 4^+} f(x) =$ 1

$\lim_{x\to 4} f(x) =$ 1

$\lim_{x\to 7^-} f(x) =$ 1

$\lim_{x\to 7^+} f(x) =$ 1

$\lim_{x\to 7} f(x) =$ 1

$\lim_{x\to 5^-} f(x) =$ 1

$\lim_{x\to 5^+} f(x) =$ 1

$\lim_{x\to 5} f(x) =$ 1

Properties of Limits

If L, M, c and k are real numbers and $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} f(x) = M$, then

1. *Sum Rule*: $\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
   $= L + M$

2. *Difference Rule:* $\lim_{x\to c}(f(x)-g(x)) = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$
   $= L - M$

3. *Product Rule:* $\lim_{x\to c}(f(x)\cdot g(x)) = \lim_{x\to c} f(x)\cdot \lim_{x\to c} g(x)$
   $= L \cdot M$

4. *Constant Multiple Rule:* $\lim_{x\to c}(kf(x)) = k\lim_{x\to c} f(x)$
   $= k \cdot L$

Properties of Limits

5. *Quotient Rule*: $\lim_{x\to c}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim_{x\to c}f(x)}{\lim_{x\to c}g(x)}$

$=\frac{L}{M}$ (provided limit of denominator is not zero)

6. *Power Rule*: $\lim_{x\to c}(f(x))^{\frac{r}{s}}=(\lim_{x\to c}f(x))^{\frac{r}{s}}$

$=L^{\frac{r}{s}}$ (provided $L^{\frac{r}{s}}$ is a real number)

Determine the limit.

Assume that $\lim_{x\to 2}h(x)=2$ *and* $\lim_{x\to 2}g(x)=-1$

a. $\lim_{x\to 2}(h(x)+g(x))=$

$\lim_{x\to 2}(h(x))+\lim_{x\to 2}g(x)=$

$2+-1=1$

b. $\lim_{x\to 2}(-4g(x))=$

4

c. $\lim_{x\to 2}(4+h(x))=$

$\lim_{x\to 2}4+\lim_{x\to 2}(h(x))=$

$4+2$

6

Limits of Polynomial Functions

Given: $f(x)=3x^2-4x+2$

Find using a calculator: $\lim_{x\to 5}f(x)=$ 57 $\lim_{x\to -2}f(x)=$ 22 $\lim_{x\to 0}f(x)=$ 2

Limits of a polynomial function may be found by substitution.

Limits of Rational Functions

If f and g are polynomial functions, then $\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c}f(x)}{\lim_{x\to c}g(x)}=\frac{f(c)}{g(c)}$ ☆

Ex: $\lim_{x\to 3}\frac{x^2-x-6}{x^2-9}=$ $\lim_{x\to 3}\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}=\frac{3+2}{3+3}=\frac{5}{6}$

Determine the limits by substitution. No Calculators.

1. $\lim_{x\to 2}x^3-2x^2+3x-4$

$2^3-2(2)^2+3(2)-4$

2

2. $\lim_{x\to 2}\frac{x^3-1}{x-1}$ $\frac{7}{1}=7$

Determine the limit by substitution then verify numerically and graphically with a calculator.

1. $\lim_{x \to 3} \frac{3x^2 - 7x - 6}{x^2 - 3x} =$

Use a calculator to determine the following limits:

$\lim_{x \to \infty} \frac{\sin x}{x} = \underline{0}$ $\quad$ $\lim_{x \to -\infty} \frac{\sin x}{x} = \underline{0}$ $\quad$ $\lim_{x \to 0} \frac{\sin x}{x} = \underline{1}$

$\lim_{x \to 0} \frac{\sin(3x)}{3x} = \underline{1}$ $\quad$ $\lim_{x \to \pm\infty} \frac{\sin(3x)}{3x} = \underline{0}$ $\quad$ $\lim_{x \to 0} \frac{x}{\sin x} = \underline{1}$ $\quad$ (?)

IMPORTANT RULE:

$\lim_{x \to 0} \frac{\sin(ax)}{ax} = \underline{1}$ $\quad$ $\lim_{x \to \infty} \frac{\sin(ax)}{ax} = \underline{0}$ $\quad$ Where a is any real number.

Determine the limits without a calculator then verify numerically and graphically with a calculator.

1. $\lim_{x \to 0} \frac{\sin 4x}{x} \cdot \frac{4}{4}$

$$\lim_{x \to 0} 4 \cdot \frac{\sin 4x}{4x}$$

$$4 \lim_{x \to 0} \frac{\sin 4x}{4x} \to 4 \cdot 1 = 4$$

2. $\lim_{x \to 0^+} \frac{|x|}{x} = 1$

3. $\lim_{x \to 0^-} \frac{|x|}{x} = -1$

4. $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{x}$

$$= \lim_{x \to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$$1 \cdot \frac{1}{1} = 1 \cdot 1 = 1$$

5. $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

6. $\lim_{x\to 0} \frac{\frac{1}{3+x} - \frac{1}{3}}{x} =$

$$\frac{(3)}{(3)} \cdot \frac{(3+x)}{(3+x)}$$

$$= \lim_{x\to 0} \frac{\frac{3}{(3+x)3} - \frac{3+x}{(3+x)3}}{x}$$

$$= \lim_{x\to 0} \frac{\frac{3-3-x}{3(3+x)}}{x}$$

$$= \lim_{x\to 0} \frac{\frac{-x}{3(3+x)}}{x}$$

$$\lim_{x\to 0} \frac{-x}{3(3+x)} \cdot \frac{1}{x}$$

$$= \frac{-1}{3(3+0)} = \frac{-1}{9}$$

7. $\lim_{x\to 0} \frac{x + \sin 4x}{x} =$

$$\lim_{x\to 0} \frac{x}{x} + \frac{\sin 4x}{x}$$

$$\lim_{x\to 0} \left(1 + \frac{\sin 4x}{x}\right)$$

$$\lim_{x\to 0} 1 + \lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{4}{4}$$

$$1 + 4 \lim_{x\to 0} \frac{\sin 4x}{4x}$$

$$1 + 4 = 5$$

Calc AB 2.1 — Name: Solutions

Day 2 and 3 (after intro)

LIMITS

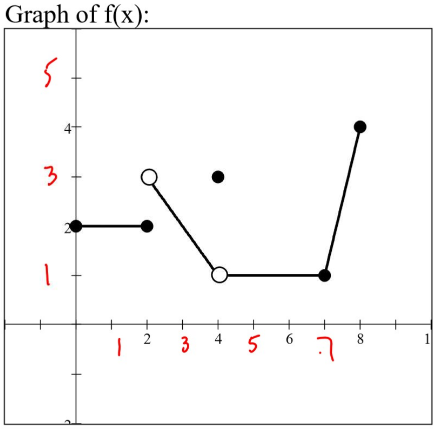
Limits give us a language for describing how the outputs (y) of a function behave as the inputs (x) approach some particular value.

Right-hand limits: $\lim\limits_{x\to c^+} f(x)$ The limit of f as x approaches c from the right (or + side)

Left-hand limits: $\lim\limits_{x\to c^-} f(x)$ The limit of f as x approaches c from the left (or − side)

Two-sided limits: $\lim\limits_{x\to c} f(x)$ Exists only if right-hand and left-hand are equal

Graph of f(x):



$\lim\limits_{x\to 2^-} f(x) =$ 2 $\qquad \lim\limits_{x\to 4^-} f(x) =$ 1

$\lim\limits_{x\to 2^+} f(x) =$ 3 $\qquad \lim\limits_{x\to 4^+} f(x) =$ 1

$\lim\limits_{x\to 2} f(x) =$ DNE $\qquad \lim\limits_{x\to 4} f(x) =$ 1

$\lim\limits_{x\to 7^-} f(x) =$ 1 $\qquad \lim\limits_{x\to 5^-} f(x) =$ 1

$\lim\limits_{x\to 7^+} f(x) =$ 1 $\qquad \lim\limits_{x\to 5^+} f(x) =$ 1

$\lim\limits_{x\to 7} f(x) =$ 1 $\qquad \lim\limits_{x\to 5} f(x) =$ 1

Properties of Limits

If L, M, c and k are real numbers and $\lim\limits_{x\to c} f(x) = \mathrm{L}$ and $\lim\limits_{x\to c} f(x) = \mathrm{M}$, then

2. *Sum Rule*:
$$\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = L + M$$

2. *Difference Rule:*
$$\lim_{x\to c}(f(x)-g(x)) = \lim_{x\to c} f(x) - \lim_{x\to c} g(x) = L - M$$

3. *Product Rule:*
$$\lim_{x\to c}(f(x)\cdot g(x)) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x) = L \cdot M$$

7. *Constant Multiple Rule:*
$$\lim_{x\to c}(kf(x)) = k\lim_{x\to c} f(x) = k \cdot L$$

Properties of Limits

8. *Quotient Rule*: $\lim_{x\to c}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim_{x\to c}f(x)}{\lim_{x\to c}g(x)}$

$=\frac{L}{M}$ (provided limit of denominator is not zero)

9. *Power Rule*: $\lim_{x\to c}(f(x))^{\frac{r}{s}}=(\lim_{x\to c}f(x))^{\frac{r}{s}}$

$=L^{\frac{r}{s}}$ (provided $L^{\frac{r}{s}}$ is a real number)

Determine the limit.

Assume that $\lim_{x\to 2}h(x)=2$ *and* $\lim_{x\to 2}g(x)=-1$

a. $\lim_{x\to 2}\left(h(x)+g(x)\right)=$ $2-1=1$

b. $\lim_{x\to 2}\left(-4g(x)\right)=$ $-4(-1)$ $=4$

c. $\lim_{x\to 2}\left(4+h(x)\right)=$ $4+2=6$

Limits of Polynomial Functions

Given: $f(x)=3x^2-4x+2$

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If f and g are polynomial functions, then $\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c}f(x)}{\lim_{x\to c}g(x)}=\frac{f(c)}{g(c)}$

Ex: $\lim_{x\to 3}\frac{x^2-x-6}{x^2-9}=$ $\frac{0}{0}$ THIS IS INDETERMINATE so factor $\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}=$

$\lim_{x\to 3}\frac{x+2}{x+3}=\frac{5}{8}$

Determine the limits by substitution. No Calculators.

1. $\lim_{x\to 2}x^3-2x^2+3x-4$

$8-8+6-4=2$

2. $\lim_{x\to 2}\frac{x^3-1}{x-1}=\frac{8-1}{2-1}=7$

Determine the limit by substitution then verify numerically and graphically with a calculator.

1. $\lim_{x \to 3} \frac{3x^2 - 7x - 6}{x^2 - 3x} =$ $\frac{27-21-6}{9-9}$ INDt. $\frac{(3x+2)\cancel{(x-3)}}{x\cancel{(x-3)}}$

$\lim_{x \to 3} \frac{3x+2}{x} = \frac{9+2}{3} = \frac{11}{3}$

Use a calculator to determine the following limits:

$\lim_{x \to \infty} \frac{\sin x}{x} =$ <u>0</u> $\lim_{x \to -\infty} \frac{\sin x}{x} =$ <u>0</u> $\lim_{x \to 0} \frac{\sin x}{x} =$ <u>1</u>

$\lim_{x \to 0} \frac{\sin(3x)}{3x} =$ <u>1</u> $\lim_{x \to \pm\infty} \frac{\sin(3x)}{3x} =$ <u>0</u> $\lim_{x \to 0} \frac{x}{\sin x} =$ <u>1</u>

IMPORTANT RULE:

$\lim_{x \to 0} \frac{\sin(ax)}{ax} =$ <u>1</u> $\lim_{x \to \infty} \frac{\sin(ax)}{ax} =$ <u>0</u> Where a is any real number.

Determine the limits without a calculator then verify numerically and graphically with a calculator.

1. $\lim_{x \to 0} \frac{\sin 4x}{x} = \frac{0}{0}$ INDT

$\frac{4}{4} \cdot \frac{\sin 4x}{x} = 4 \frac{\sin 4x}{4x}$

$\lim_{x \to 0} 4 \cdot \frac{\sin 4x}{4x} = 4 \cdot 1 = 4$

2. $\lim_{x \to 0^+} \frac{|x|}{x}$ note, $|x|$ for $x \geq 0$ so

$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$

3. $\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$

4. $\lim_{x \to 0} \frac{\tan x}{x} = \frac{0}{0}$ INDT

$\lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = \lim_{x \to \infty} \frac{\sin x}{x} \cdot \lim_{x \to \infty} \frac{1}{\cos}$

$= 1 \cdot 1 = 1$

5. $\lim_{x \to 0} \dfrac{\frac{1}{3+x} - \frac{1}{3}}{x} =$ $\frac{0}{0}$ INDT

$$\frac{\frac{3}{3(3+x)} - \frac{3+x\,(1)}{3(3+x)}}{x}$$

$$\frac{\frac{3-3-x}{9+3x}}{x}$$

$$\frac{\frac{-x}{9+3x}}{x} = \frac{-1}{9+3x}$$

$$\lim_{x \to 0} \frac{-1}{9+3x} = -\frac{1}{9}$$

6. $\lim_{x \to 0} \dfrac{x + \sin 4x}{x} =$ $\frac{0+0}{0}$ INDT

$$\frac{x}{x} + \frac{\sin 4x}{x}$$

$$1 + 4 \cdot \frac{\sin 4x}{4x}$$

$$\lim_{x \to 0} \left(1 + 4\,\frac{\sin 4x}{4x}\right) = 1 + 4 \cdot 1 = 5$$