

NO CALCULATOR

Find the following limits:

1. $\lim_{x \rightarrow 5} \frac{25 - x^2}{x - 5} =$

$\lim_{x \rightarrow 5} \frac{(5-x)(5+x)}{x-5}$
 $\lim_{x \rightarrow 5} \frac{-(x-5)(x+5)}{(x-5)} = -10$

4. $\lim_{x \rightarrow -1^-} \frac{1}{1+x^2} = \infty$

7. $\lim_{x \rightarrow 2} \frac{x^3 - 2x^2 + x - 2}{x - 2}$

$$\begin{array}{r} 2 \overline{) 1 - 2 \quad 1 - 2} \\ \underline{2 \quad 0 \quad 2} \\ 1 \quad 0 \quad 1 \quad 0 \end{array}$$

$\lim_{x \rightarrow 2} \frac{(x-2)(x^2+1)}{(x-2)} = 5$

10. $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{(\sqrt{4+h} + 2)}{(\sqrt{4+h} + 2)}$

$\lim_{h \rightarrow 0} \frac{4+h-4}{h(\sqrt{4+h} + 2)}$

$\lim_{h \rightarrow 0} \frac{h}{h(\sqrt{4+h} + 2)}$

$\frac{1}{4}$

2. $\lim_{x \rightarrow \pi} \ln \left(\sin \frac{x}{2} \right) =$

$= \ln \left(\sin \frac{\pi}{2} \right)$

$= \ln 1$

$= 0$

5. $\lim_{x \rightarrow 2^+} \frac{x-2}{|x-2|} = 1$

8. $\lim_{x \rightarrow -3} \frac{x^2 + 7x + 12}{x^2 - 9}$

$\lim_{x \rightarrow -3} \frac{(x+4)(x+3)}{(x-3)(x+3)}$

$\frac{1}{-6}$

11. $\lim_{x \rightarrow 0} \frac{\frac{2}{3+x} - \frac{2}{3}}{x}$

$\lim_{x \rightarrow 0} \frac{6 - 2(3+x)}{3(3+x)x}$

$\lim_{x \rightarrow 0} \frac{6 - 6 - 2x}{3x(3+x)}$

$\lim_{x \rightarrow 0} \frac{-2}{3(3+x)}$

$= -\frac{2}{9}$

3. $\lim_{x \rightarrow 0} x\sqrt{4-x}$

$0 \cdot \sqrt{4-0}$

0

6. $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

$\lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$

$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$

$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$
 $1 \cdot 1$
 $= 1$

9. $\lim_{x \rightarrow -2} \frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}}$

$\lim_{x \rightarrow -2} \frac{2(x+2) - 4}{5(x+2) + 10}$

$\lim_{x \rightarrow -2} \frac{2x + 4 - 4}{5x + 10 + 10}$

$= \frac{-4}{-10 + 10 + 10} = -\frac{4}{10} = -\frac{2}{5}$

$-\frac{4}{10} = -\frac{2}{5}$

12. $\lim_{x \rightarrow 0} \frac{\sin x}{2x^2 - x}$

$\lim_{x \rightarrow 0} \frac{\sin x}{x(2x-1)}$

$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{2x-1}$

$1 \cdot \frac{1}{-1} = -1$

$$13. f(x) = \begin{cases} x & x < 0 \\ -x^2 + 2 & 0 \leq x < 2 \\ x - 4 & x > 2 \end{cases}$$

$-4 + 2 = -2$
 $2 - 4 = -2$

a. $\lim_{x \rightarrow 0^+} f(x) = 2$

b. $\lim_{x \rightarrow 2} f(x) = -2$

c. Is $f(x)$ continuous at $x=2$? Justify
 no b/c $f(2)$ dne

$$14. g(x) = \begin{cases} |x| & -1 < x \leq 1 \\ 4 & x = -1 \\ x^2 & 1 < x \leq 4 \end{cases}$$

a. Is $g(x)$ continuous at $x=1$? Justify

yes b/c $\lim_{x \rightarrow 1} f(x) = f(1) = 1$