11.1 Day 1

Friday, May 24, 2019 7:39 AM

opener

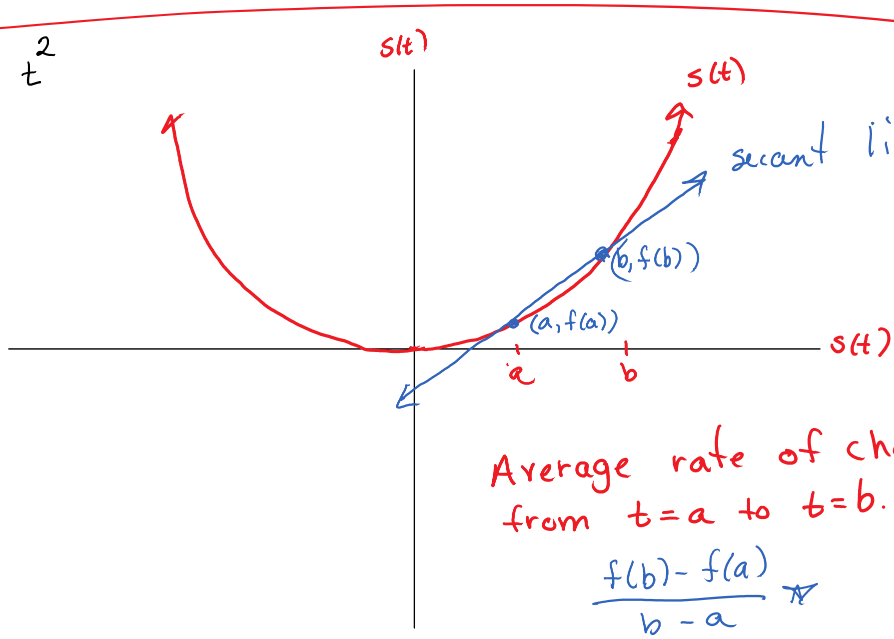
a. $\lim_{x \to \infty} \frac{2x^2 - x - 3}{x^2 - 1} = 2$

b. $\lim_{x \to \infty} \frac{3x - 1}{5x^3 + 10} = 0$

c. $f(x) = \begin{cases} x^2 - 4 & x \le 2 \quad L \\ 5x + 2 & x > 2 \quad R \end{cases}$

$\lim_{x \to 2^-} f(x) = 0$ $\lim_{x \to 2^+} f(x) = 12$ $f(2) = 0$

$s(t) = t^2$



Average rate of change from $t = a$ to $t = b$.

$\frac{f(b) - f(a)}{b - a}$ ☆

$s(t) =$ position of a particle in feet $t =$ seconds

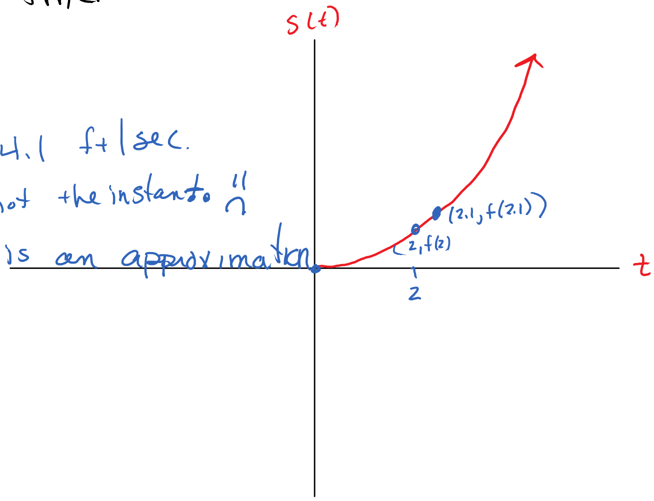
a. find the average velocity of the particle from $t = 1$ to $t = 3$.

$$\frac{s(3) - s(1)}{3 - 1} = \frac{3^2 - 1^2}{2} = \frac{8}{2} = 4 \text{ ft/sec}$$

b. find the average velocity of the first 2 seconds.

$$\frac{S(2)-S(0)}{2-0} = 2 \text{ ft/sec}$$

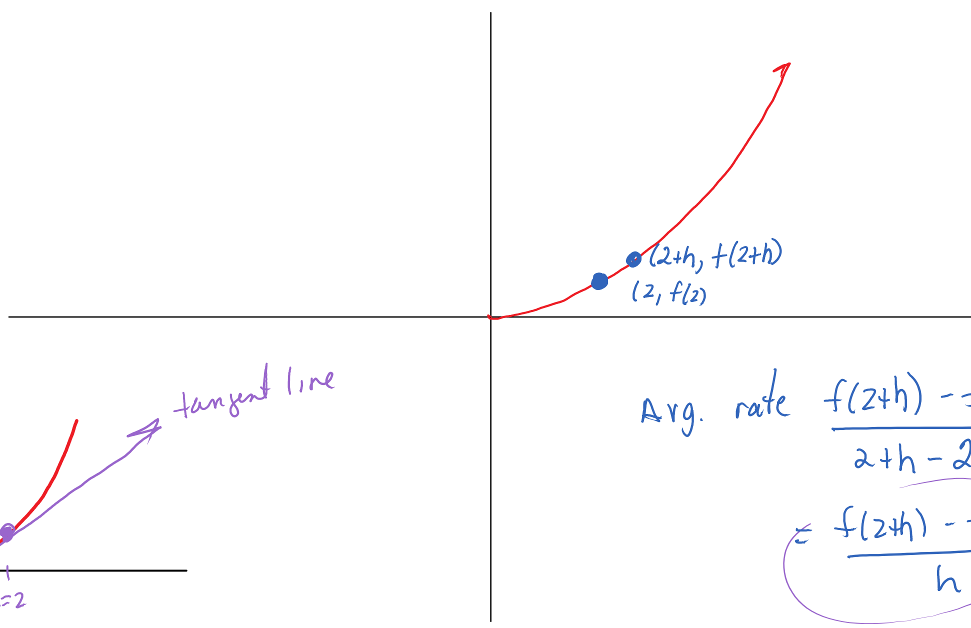
c. How can we find the instantaneous velocity @ $t = 2$ secs?

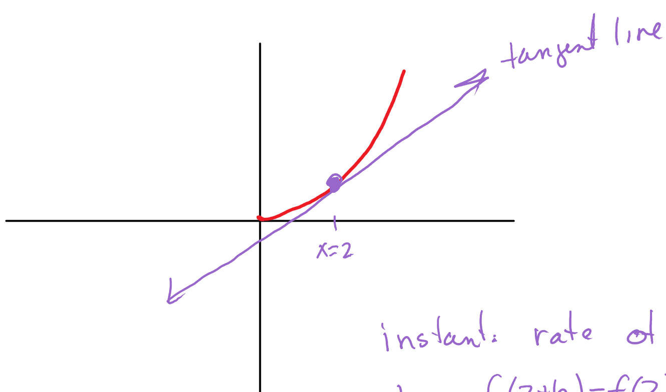


$$\frac{f(2.1)-f(2)}{2.1-2} = 4.1 \text{ ft/sec.}$$

not the instant. "

It is an approximation





Avg. rate $\frac{f(2+h)-f(2)}{2+h-2}$

$= \frac{f(2+h)-f(2)}{h}$ slope of the secant line.

instant. rate of change @ $x=2$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \text{slope of the tangent line @ } x=2$$

= instantaneous rate of change @ $x=2$

= $s'(2)$ the derivative of $s(t)$ @ $t=2$

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$$\lim_{h\to 0} \frac{(2+h)^2 - 2^2}{h} = \lim_{h\to 0} \frac{\cancel{4}+4h+h^2-\cancel{4}}{h} = \lim_{h\to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = 4$$

Definition of a derivative at a point $x=a$

$$\star\ f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$ provided the limit exists

Definition of the derivative of a function.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

example: find $f'(x)$ given $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

$$f'(x) = 2x$$

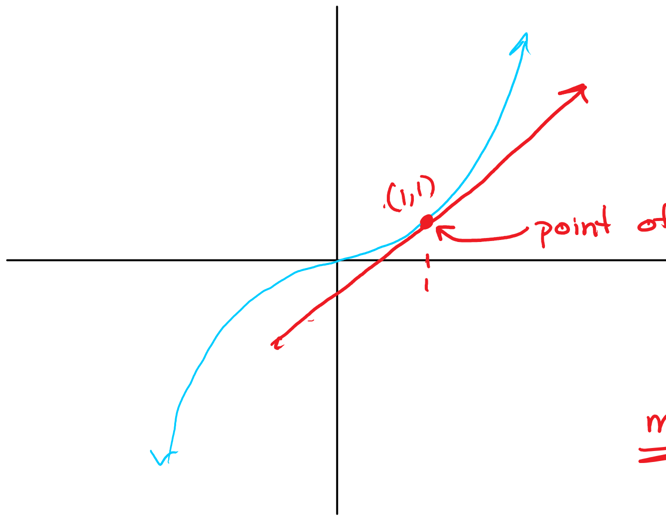
Alternate defn. of a derivative (slope of a tangent line) at a point.

Recall $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$\Big\}$ provided the limit exists

Find the equation of the tangent line to $f(x)=x^3$ @ $x=1$.



point of tangency (p.o.t)

point of tangency

$(1, f(1)) \Rightarrow (1,1)$

slope @ $x=1$

$$m = f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$$

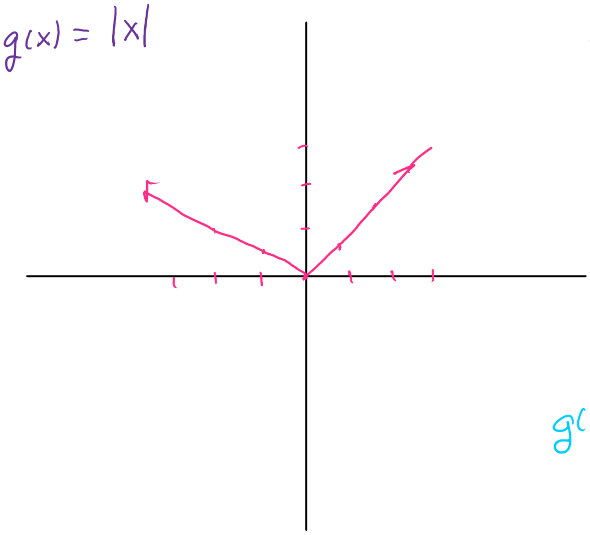
$$= \lim_{x\to 1} \frac{x^3-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$$

$$m = f'(1) = 1^2+1+1 = 3$$

$(1,1)$ $m=3$

$$\boxed{y-1 = 3(x-1)}$$

$g(x) = |x|$



Estimate the slope of $g(x)$ at:

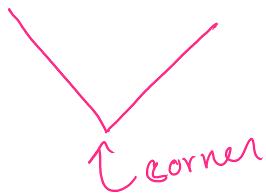
a. $x=1$ $m=1$

b. $x=-1$ $m=-1$

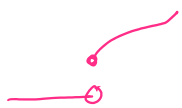
c. $x=0$ dne

$g(x)$ is not differentiable @ $x=0$

A function is not differentiable:



any discontinuities



vertical tangent